NSW GOVERNMENT

NSW Education Standards Authority

2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–15)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

2500

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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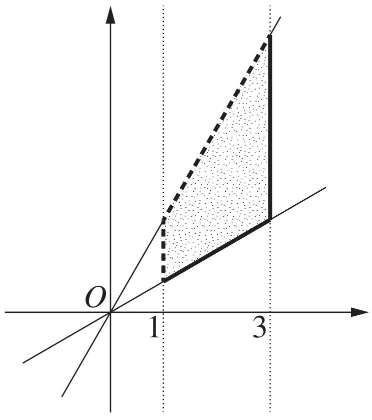
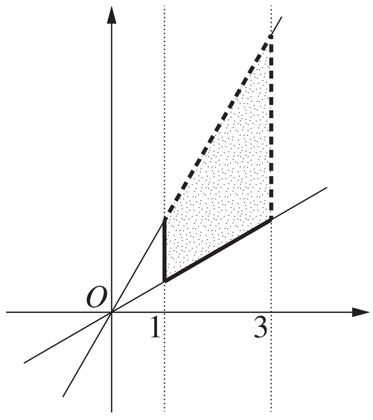
1 Let R be the region in the complex plane defined by $1 < \text{Re}(z) \le 3$ and $\dfrac{\pi}{6} \le \text{Arg}(z) < \dfrac{\pi}{3}$.

Which diagram best represents the region R?

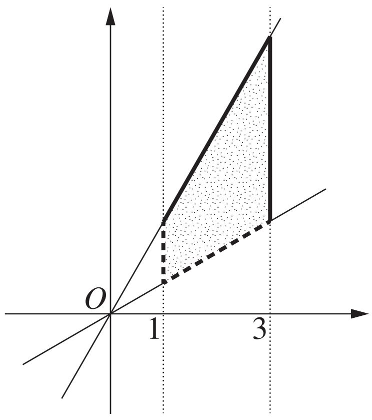
A.



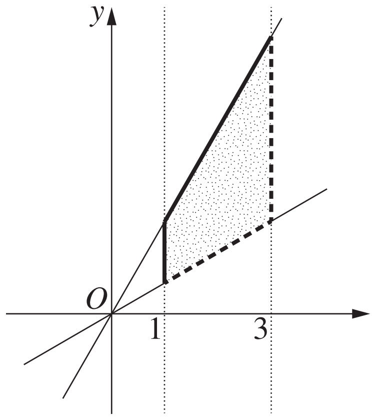
B.



C.



D.



2 The following proof aims to establish that $-4 = 0$.

$$\begin{array}{lll} \text{Let} & a = -4 & \\ \Rightarrow & a^2 = 16 \text{ and } 4a + 4 = -12 & \text{Line 1} \\ \Rightarrow & a^2 + 4a + 4 = 4 & \text{Line 2} \\ \Rightarrow & (a+2)^2 = 2^2 & \text{Line 3} \\ \Rightarrow & a + 2 = 2 & \text{Line 4} \\ \Rightarrow & a = 0 & \end{array}$$

At which line is the implication incorrect?

A. Line 1

B. Line 2

C. Line 3

D. Line 4

3 Let A, B, P be three points in three-dimensional space with $A \neq B$.

Consider the following statement.

If P is on the line AB, then there exists a real number λ such that $\overrightarrow{AP} = \lambda \overrightarrow{AB}$.

Which of the following is the contrapositive of this statement?

A. If for all real numbers λ, $\overrightarrow{AP} = \lambda \overrightarrow{AB}$, then P is on the line AB.

B. If for all real numbers λ, $\overrightarrow{AP} \neq \lambda \overrightarrow{AB}$, then P is not on the line AB.

C. If there exists a real number λ such that $\overrightarrow{AP} = \lambda \overrightarrow{AB}$, then P is on the line AB.

D. If there exists a real number λ such that $\overrightarrow{AP} \neq \lambda \overrightarrow{AB}$, then P is not on the line AB.

4 Of the following expressions, which one need NOT contain a term involving a logarithm in its anti-derivative?

A. $\dfrac{x+2}{x^2+4x+5}$

B. $\dfrac{x+2}{x^2-4x-5}$

C. $\dfrac{x-1}{x^3-x^2+x-1}$

D. $\dfrac{x+1}{x^3-x^2+x-1}$

5 If $\int_a^x f(t)\,dt = g(x)$, which of the following is a primitive of $f(x)g(x)$?

A. $\frac{1}{2}[f(x)]^2$

B. $\frac{1}{2}[f'(x)]^2$

C. $\frac{1}{2}[g(x)]^2$

D. $\frac{1}{2}[g'(x)]^2$

6 It is known that a particular complex number z is NOT a real number.

Which of the following could be true for this number z?

A. $\overline{z} = iz$

B. $\overline{z} = |z^2|$

C. $\text{Re}(iz) = \text{Im}(z)$

D. $\text{Arg}\left(z^3\right) = \text{Arg}(z)$

7 Consider the statement P.

P: For all integers $n \geq 1$, if n is a prime number then $\dfrac{n(n+1)}{2}$ is a prime number.

Which of the following is true about this statement and its converse?

A. The statement P and its converse are both true.

B. The statement P and its converse are both false.

C. The statement P is true and its converse is false.

D. The statement P is false and its converse is true.

8 As a projectile of mass m kilograms travels through air, it experiences a frictional force. The magnitude of this force is proportional to the square of the speed v of the projectile. The constant of proportionality is the positive number k. The position of the particle at time t is denoted by $\begin{pmatrix} x \\ y \end{pmatrix}$. The acceleration due to gravity is $g \text{ m s}^{-2}$.

Based on Newton's laws of motion, which equation models the motion of this projectile?

A. $\begin{pmatrix} 0 \\ -mg \end{pmatrix} + kv\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = m\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$

B. $\begin{pmatrix} 0 \\ -mg \end{pmatrix} - kv\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = m\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$

C. $\begin{pmatrix} 0 \\ -mg \end{pmatrix} + kv^2\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = m\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$

D. $\begin{pmatrix} 0 \\ -mg \end{pmatrix} - kv^2\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = m\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$

9 Let A and B be two distinct points in three-dimensional space. Let M be the midpoint of AB.

Let S_1 be the set of all points P such that $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$.

Let S_2 be the set of all points N such that $\left|\overrightarrow{AN}\right| = \left|\overrightarrow{MN}\right|$.

The intersection of S_1 and S_2 is the circle S.

What is the radius of the circle S?

A. $\dfrac{\left|\overrightarrow{AB}\right|}{2}$

B. $\dfrac{\left|\overrightarrow{AB}\right|}{4}$

C. $\dfrac{\sqrt{3}\left|\overrightarrow{AB}\right|}{2}$

D. $\dfrac{\sqrt{3}\left|\overrightarrow{AB}\right|}{4}$

10 A particle is moving vertically in a resistive medium under the influence of gravity. The resistive force is proportional to the velocity of the particle.

The initial speed of the particle is NOT zero.

Which of the following statements about the motion of the particle is always true?

A. If the particle is initially moving downwards, then its speed will increase.

B. If the particle is initially moving downwards, then its speed will decrease.

C. If the particle is initially moving upwards, then its speed will eventually approach a terminal speed.

D. If the particle is initially moving upwards, then its speed will not eventually approach a terminal speed.

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet

(a) Express $\dfrac{3-i}{2+i}$ in the form $x+iy$, where x and y are real numbers. **2**

(b) Evaluate $\displaystyle\int \sin^3 2x \cos 2x \, dx$. **2**

(c) (i) Write the complex number $-\sqrt{3}+i$ in exponential form. **2**

(ii) Hence, find the exact value of $\left(-\sqrt{3}+i\right)^{10}$ giving your answer in the form $x+iy$. **2**

(d) A triangle is formed in three-dimensional space with vertices $A(1,-1,2)$, $B(0,2,-1)$ and $C(2,1,1)$. **3**

Find the size of $\angle ABC$, giving your answer to the nearest degree.

(e) Let ℓ_1 be the line with equation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\lambda \in \mathbb{R}$. **2**

The line ℓ_2 passes through the point $A(-6, 5)$ and is parallel to ℓ_1.

Find the equation of the line ℓ_2 in the form $y = mx + c$.

(f) Using the substitution $t = \tan\dfrac{x}{2}$, find **3**

$$\int \frac{dx}{1+\cos x - \sin x}.$$

Question 12 (15 marks) Use the Question 12 Writing Booklet

(a) For real numbers a, $b \geq 0$ prove that $\dfrac{a+b}{2} \geq \sqrt{ab}$. **2**

(b) A particle is moving in a straight line with acceleration $a = 12 - 6t$. The particle starts from rest at the origin. **3**

What is the position of the particle when it reaches its maximum velocity?

(c) A particle with mass 1 kg is moving along the x-axis. Initially, the particle is at the origin and has speed u m s^{-1} to the right. The particle experiences a resistive force of magnitude $v + 3v^2$ newtons, where v m s^{-1} is the speed of the particle after t seconds. The particle is never at rest.

(i) Show that $\dfrac{dv}{dx} = -(1 + 3v)$. **1**

(ii) Hence, or otherwise, find x as a function of v. **2**

(d) Using partial fractions, evaluate $\displaystyle\int_2^n \frac{4+x}{(1-x)(4+x^2)}\,dx$, giving your answer in the form $\dfrac{1}{2}\ln\left(\dfrac{f(n)}{8(n-1)^2}\right)$, where $f(n)$ is a function of n. **4**

(e) Given the complex number $z = e^{i\theta}$, show that $w = \dfrac{z^2 - 1}{z^2 + 1}$ is purely imaginary. **3**

Question 13 (14 marks) Use the Question 13 Writing Booklet

(a) Prove that for all integers n with $n \geq 3$, if $2^n - 1$ is prime, then n cannot be even. **3**

(b) The numbers a_n, for integers $n \geq 1$, are defined as **4**

$$a_1 = \sqrt{2}$$
$$a_2 = \sqrt{2 + \sqrt{2}}$$
$$a_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \text{ and so on.}$$

These numbers satisfy the relation $a_{n+1}^2 = 2 + a_n$, for $n \geq 1$. (Do NOT prove this.)

Use mathematical induction to prove that $a_n = 2\cos\frac{\pi}{2^{n+1}}$, for all integers $n \geq 1$.

(c) Consider the equation $z^5 + 1 = 0$, where z is a complex number.

(i) Solve the equation $z^5 + 1 = 0$ by finding the 5th roots of -1. **2**

(ii) Show that if z is a solution of $z^5 + 1 = 0$ and $z \neq -1$, then $u = z + \frac{1}{z}$ is a solution of $u^2 - u - 1 = 0$. **2**

(iii) Hence find the exact value of $\cos\frac{3\pi}{5}$. **3**

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) (i) The two non-parallel vectors $\vec{u}$ and $\vec{v}$ satisfy $\lambda \vec{u} + \mu \vec{v} = \vec{0}$ for some real numbers λ and μ. **2**

Show that $\lambda = \mu = 0$.

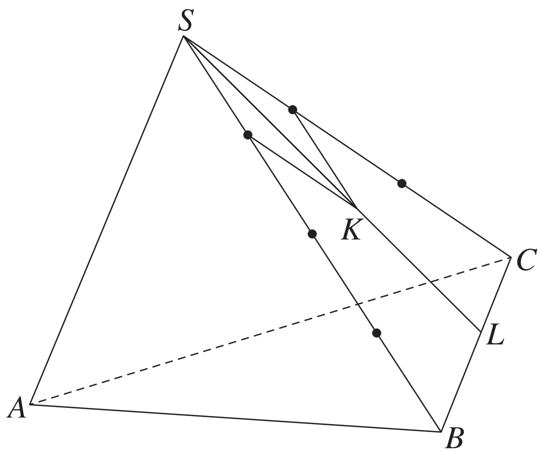
(ii) The two non-parallel vectors $\vec{u}$ and $\vec{v}$ satisfy $\lambda_1 \vec{u} + \mu_1 \vec{v} = \lambda_2 \vec{u} + \mu_2 \vec{v}$ for some real numbers λ_1, λ_2, μ_1 and μ_2. **1**

Using part (i), or otherwise, show that $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$.

The diagram below shows the tetrahedron with vertices A, B, C and S.

The point K is defined by $\overrightarrow{SK} = \frac{1}{4}\overrightarrow{SB} + \frac{1}{3}\overrightarrow{SC}$, as shown in the diagram.

The point L is the point of intersection of the straight lines SK and BC.



(iii) Using part (ii), or otherwise, determine the position of L by showing that $\overrightarrow{BL} = \frac{4}{7}\overrightarrow{BC}$. **2**

(iv) The point P is defined by $\overrightarrow{AP} = -6\overrightarrow{AB} - 8\overrightarrow{AC}$. **2**

Does P lie on the line AL? Justify your answer.

Question 14 continues on page 11

(b) Let $J_n = \int_0^1 x^n e^{-x}\, dx$, where n is a non-negative integer.

(i) Show that $J_0 = 1 - \dfrac{1}{e}$. **1**

(ii) Show that $J_n \le \dfrac{1}{n+1}$. **2**

(iii) Show that $J_n = nJ_{n-1} - \dfrac{1}{e}$, for $n \ge 1$. **2**

(iv) Using parts (i) and (iii), show by mathematical induction, or otherwise, that for all $n \ge 0$, **2**

$$J_n = n! - \frac{n!}{e}\sum_{r=0}^{n}\frac{1}{r!}.$$

(v) Using parts (ii) and (iv) prove that $e = \lim_{n\to\infty}\sum_{r=0}^{n}\frac{1}{r!}$. **1**

End of Question 14

Please turn over

Question 15 (15 marks) Use the Question 15 Writing Booklet

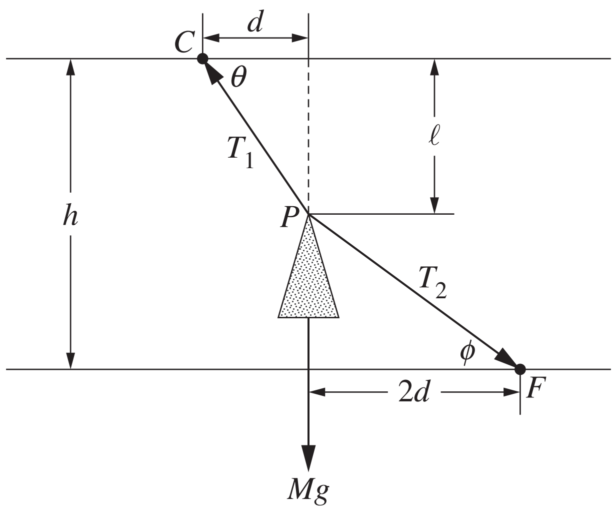
(a) A machine is lifted from the floor of a room using two ropes. The two ropes ensure that the horizontal components of the forces are balanced at all times. It is assumed that at all times the machine moves vertically upwards at a constant velocity.

The machine is located in a room with height h metres.

One of the ropes is attached to the point P on the machine and to the fixed point C on the ceiling of the room. The point C is a distance d metres to the left of P. Let the vertical distance from P to the ceiling be ℓ metres and let θ be the angle this rope makes with the horizontal.

The other rope is attached to the point P and to the fixed point F on the floor of the room. The point F is a distance $2d$ metres to the right of P. Let ϕ be the angle this rope makes with the horizontal.

Let the tension in the first rope be T_1 newtons, the tension in the second rope be T_2 newtons, the mass of the machine be M kilograms and the acceleration due to gravity be g m s^{-2}.



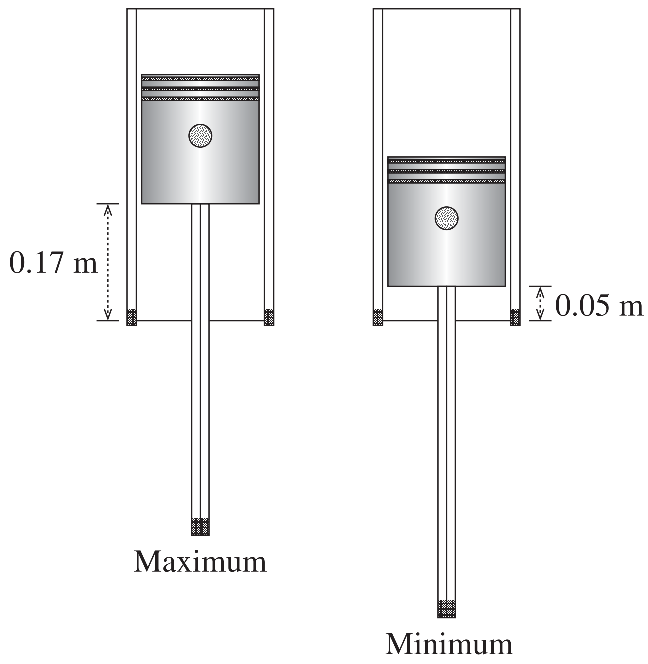
(i) By considering horizontal and vertical components of the forces at P, show that **3**

$$\tan\theta = \tan\phi + \frac{Mg}{T_2\cos\phi}.$$

(ii) Hence, or otherwise, show that the point P cannot be lifted to a position $\frac{2h}{3}$ metres above the floor. **2**

Question 15 continues on page 13

(b) The diagrams show two positions of a single piston in the cylinder chamber of a motorcycle. The piston moves vertically, in simple harmonic motion, between a maximum height of 0.17 metres and a minimum height of 0.05 metres. **3**



The mass of the piston is 0.8 kg. The piston completes 40 cycles per second.

What is the resultant force on the piston, in newtons, that produces the maximum acceleration of the piston? Give your answer correct to the nearest newton.

(c) Using the substitution $x = \tan^2\theta$, evaluate **4**

$$\int_0^1 \sin^{-1}\sqrt{\frac{x}{1+x}}\,dx.$$

(d) The complex number z satisfies $\left|z - \frac{4}{z}\right| = 2$. **3**

Using the triangle inequality, or otherwise, show that $|z| \leq \sqrt{5} + 1$.

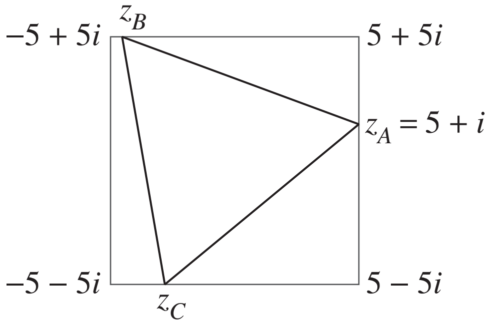
End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet

(a) A square in the Argand plane has vertices **4**

$$5 + 5i, \qquad 5 - 5i, \qquad -5 - 5i \qquad \text{and} \qquad -5 + 5i.$$

The complex numbers $z_A = 5 + i$, z_B and z_C lie on the square and form the vertices of an equilateral triangle, as shown in the diagram.



NOT TO SCALE

Find the exact value of the complex number z_B.

(b) A projectile of mass M kg is launched vertically upwards from a horizontal plane with initial speed v_0 m s^{-1} which is less than 100 m s^{-1}. The projectile experiences a resistive force which has magnitude $0.1Mv$ newtons, where v m s^{-1} is the speed of the projectile. The acceleration due to gravity is 10 m s^{-2}. **4**

The projectile lands on the horizontal plane 7 seconds after launch.

Find the value of v_0, correct to 1 decimal place.

Question 16 continues on page 15

(c) It is given that for positive numbers $x_1, x_2, x_3, \ldots, x_n$ with arithmetic mean A,

$$\frac{x_1 \times x_2 \times x_3 \times \cdots \times x_n}{A^n} \le 1. \qquad \text{(Do NOT prove this.)}$$

Suppose a rectangular prism has dimensions a, b, c and surface area S.

(i) Show that $abc \le \left(\frac{S}{6}\right)^{\frac{3}{2}}$. **2**

(ii) Using part (i), show that when the rectangular prism with surface area S is a cube, it has maximum volume. **2**

(d) Find all the complex numbers z_1, z_2, z_3 that satisfy the following three conditions simultaneously. **3**

$$\begin{cases} |z_1| = |z_2| = |z_3| \\ z_1 + z_2 + z_3 = 1 \\ z_1 z_2 z_3 = 1 \end{cases}$$

End of paper

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2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

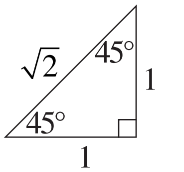
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

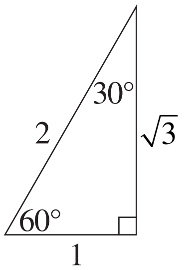


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

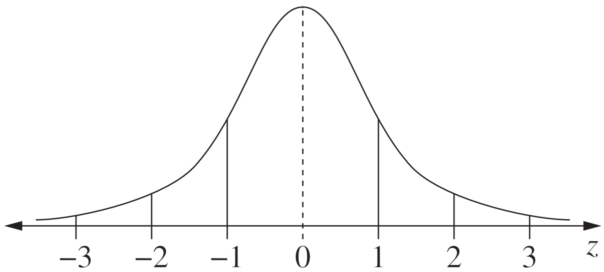
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^ne^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$